Mathematics: analysis and approaches
Standard level
Paper 2

31 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

    Consider the function defined by $f(x) = x^2 - 8x$. The graph of f passes through the point $A(3, -15)$.

    (a) (i) Find the gradient of the tangent to the graph of f at the point A.

         (ii) Hence, write down the gradient of the normal to the graph of f at point A. [3]

    (b) Write down the equation of the normal to the graph of f at point A. [1]

    The normal to the graph of f at point A intersects the graph of f again at a second point B.

    (c) Find the coordinates of B. [3]

2. [Maximum mark: 6]

The following diagram shows a pyramid with vertex V and rectangular base OABC.

Point B has coordinates $(6, 8, 0)$, point C has coordinates $(6, 0, 0)$ and point V has coordinates $(3, 4, 9)$.

diagram not to scale
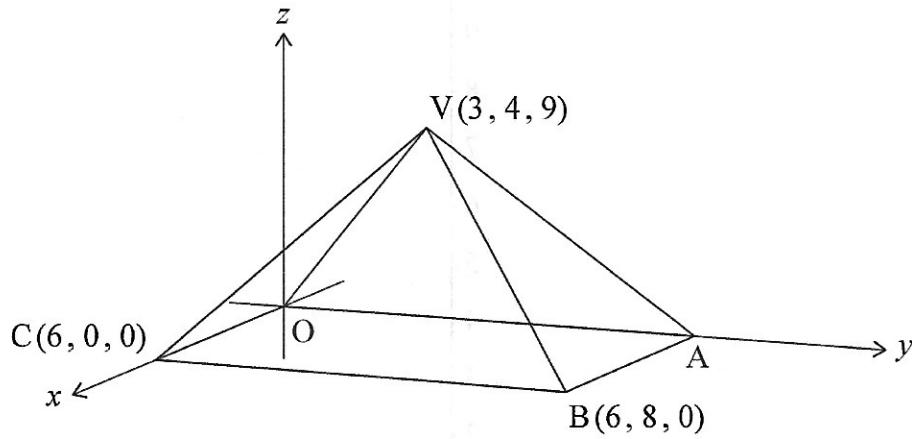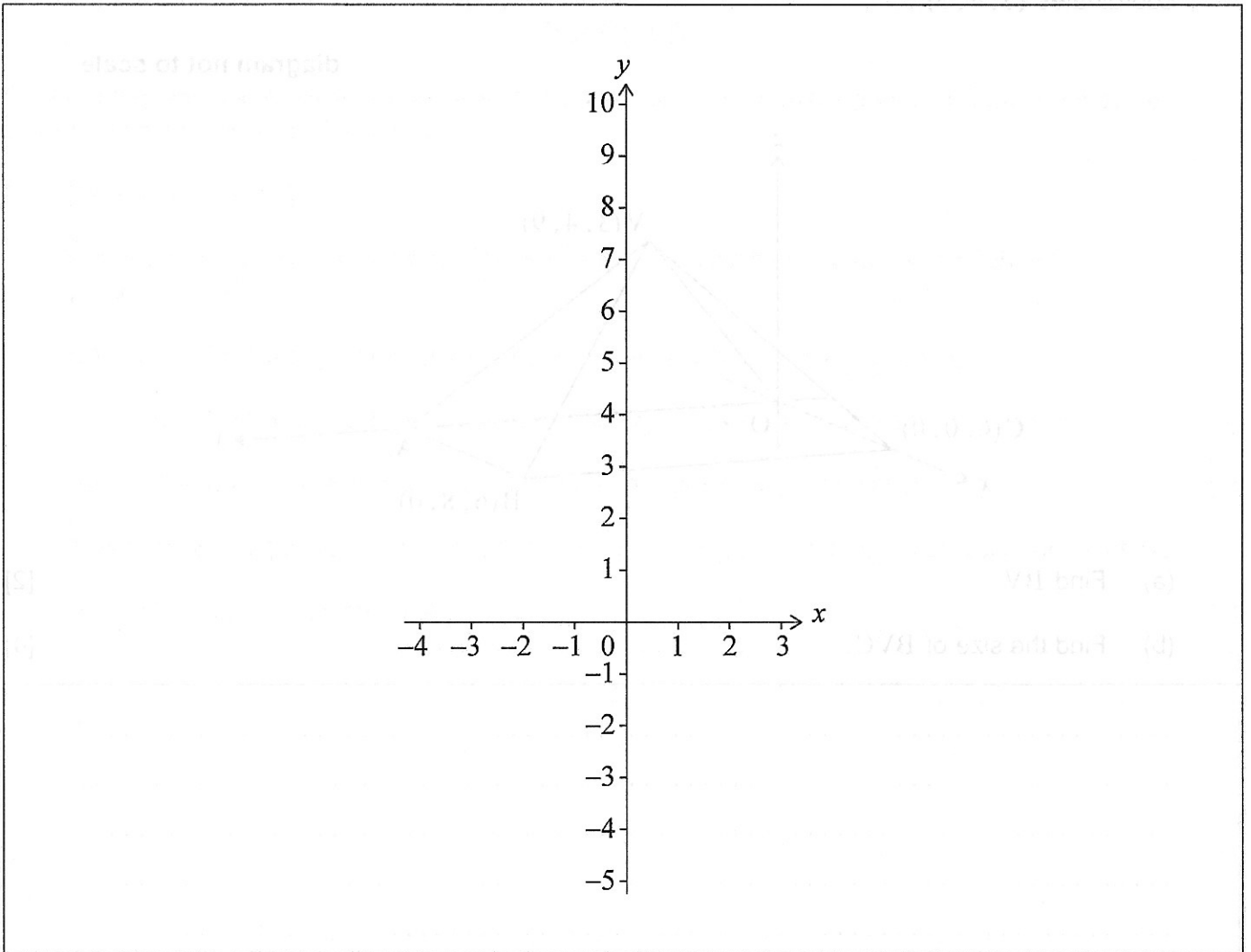

(a) Find BV. [2]

(b) Find the size of $B\hat{V}C$. [4]

3. [Maximum mark: 5]

Consider the function $f(x) = e^x - 3x - 4$.

(a) On the following axes, sketch the graph of f for $-4 \leq x \leq 3$. [3]

The function g is defined by $g(x) = e^{2x} - 6x - 7$.

(b) The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k, followed by a vertical translation of c units.
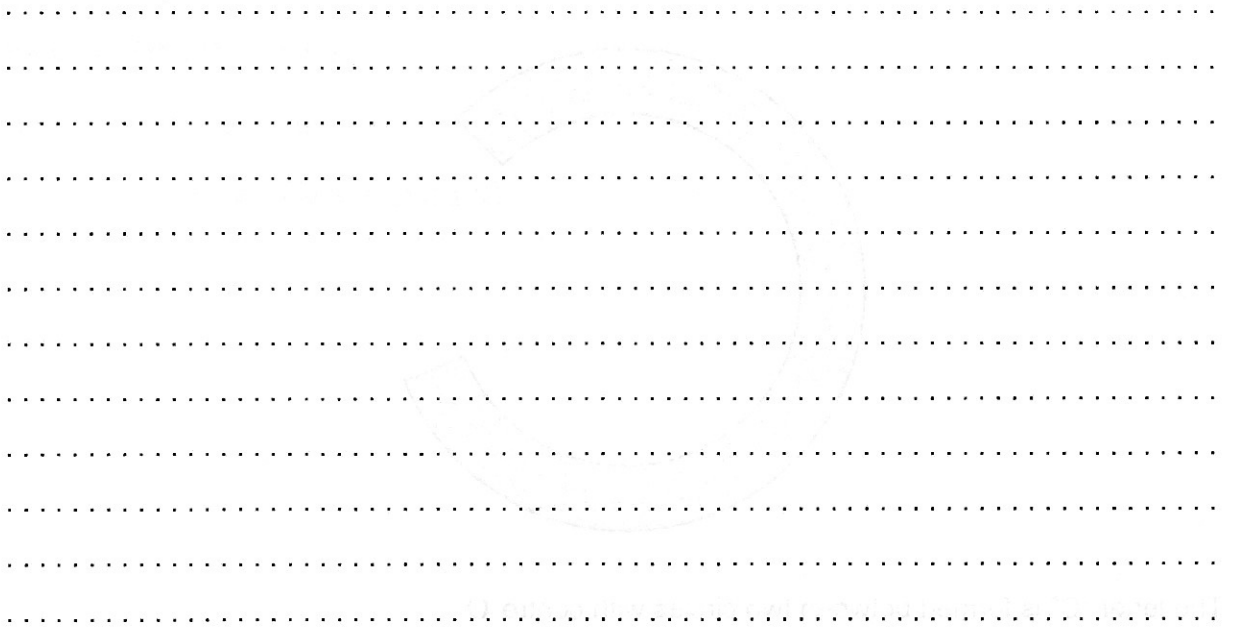
Find the value of k and the value of c. [2]

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(Question 3 continued)

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4.   [Maximum mark: 7]

A company is designing a new logo in the shape of a letter "C".
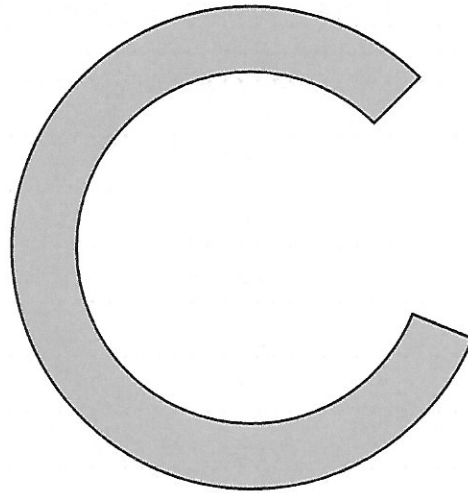
The letter "C" is formed between two circles with centre O.

The point A lies on the circumference of the inner circle with radius r cm, where $r < 10$.

The point B lies on the circumference of the outer circle with radius 10 cm.
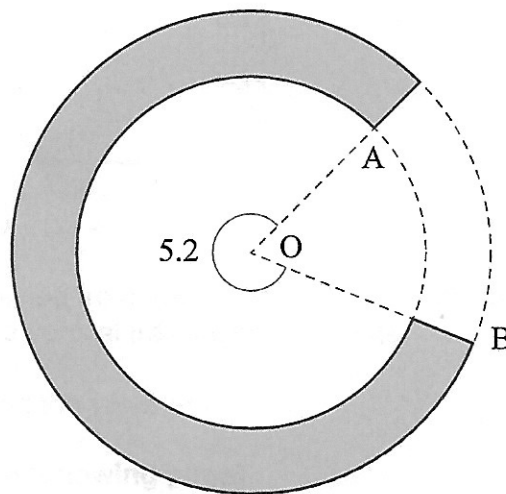
The reflex angle $A\hat{O}B$ is 5.2 radians. The letter "C" is shown by the shaded area in the following diagram.

diagram not to scale

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(Question 4 continued)

(a)   Show that the area of the "C" is given by $260 - 2.6r^2$.   [2]

The area of the "C" is $64\,\text{cm}^2$.

(b)   (i)   Find the value of r.

(ii)   Find the perimeter of the "C".   [5]

5. [Maximum mark: 5]

A particle moves along a straight line. Its displacement, s metres, from a fixed point O after time t seconds is given by $s(t) = 4.3 \sin\left(\sqrt{3t+5}\right)$, where $0 \leq t \leq 10$.

The particle first comes to rest after q seconds.

(a) Find the value of q. [2]

(b) Find the total distance that the particle travels in the first q seconds. [3]

6. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X, where $a, k \in \mathbb{R}^+$.

x	1	2	3	4
$P(X = x)$	k	k^2	a	k^3

Given that $E(X) = 2.3$, find the value of a.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

The solid shown in the following diagram is comprised of a cylinder and two hemispheres. The cylinder has height h cm and radius x cm. The hemispheres fit exactly onto either end of the cylinder.
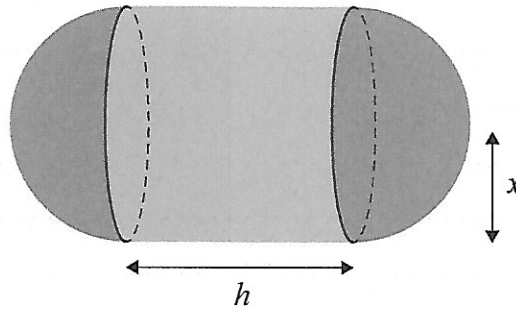
The volume of the cylinder is 45 cm^3.

(a) Show that the total surface area, $S \text{ cm}^2$, of the solid is given by $S = \dfrac{90}{x} + 4\pi x^2$. [3]

The total surface area of the solid has a local maximum or a local minimum value when $x = a$.

(b) (i) Find an expression for $\dfrac{\mathrm{d}S}{\mathrm{d}x}$.

(ii) Hence, find the **exact** value of a. [5]

(c) (i) Find an expression for $\dfrac{\mathrm{d}^2 S}{\mathrm{d}x^2}$.

(ii) Use the second derivative of S to justify that S is a minimum when $x = a$.

(iii) Find the minimum surface area of the solid. [6]

Do **not** write solutions on this page.

8.   [Maximum mark: 15]

Give your answers to parts (a)(ii), (c)(i) and (d) correct to two decimal places.

Daniela and Sorin have each recently received some money. Daniela won a cash prize and Sorin received an inheritance.

Daniela had two options to choose from to receive her winnings. In both options she receives a payment on the first day of each month for three years.

Option A   Each payment is $5500.

Option B   The first payment is $2000. In each month which follows, the payment is 6% more than the previous month.

(a)   Find the total amount Daniela would receive if she chooses

       (i)   Option A;

       (ii)   Option B.   [5]

Sorin received an inheritance of $120 000. Sorin invested his inheritance in an account that pays a nominal annual interest rate of 4% per annum, compounded monthly. The interest is added on the last day of each month.

(b)   Write down an expression for the value of Sorin's investment after n years.   [1]

Daniela chose Option B and received her first payment on 1st January 2023. Sorin invested his inheritance on the same day.

(c)   (i)   Find the **total** value of Daniela's winnings and Sorin's investment on the last day of the sixth month.

       (ii)   Find the minimum number of complete months before the total value of Daniela's winnings and Sorin's investment is at least $250 000.   [6]

At the end of the three years, Daniela invested $40 000 for a further six years in a second account that pays a nominal interest rate of r% per annum compounded quarterly.

(d)   Find the value of r if this investment grows to $53 000 after six years.   [3]

Do **not** write solutions on this page.

9.   [Maximum mark: 16]

A farmer is growing a field of wheat plants. The height, H cm, of each plant can be modelled by a normal distribution with mean μ and standard deviation σ.

It is known that $P(H < 94.6) = 0.288$ and $P(H > 98.1) = 0.434$.

(a)   Find the probability that the height of a randomly selected plant is between 94.6 cm and 98.1 cm.   [2]

(b)   Find the value of μ and the value of σ.   [5]

The farmer measures 100 randomly selected plants. Any plant with a height greater than 98.1 cm is considered ready to harvest. Heights of plants are independent of each other.

(c)   (i)   Find the probability that exactly 34 plants are ready to harvest.

       (ii)   Given that fewer than 49 plants are ready to harvest, find the probability that exactly 34 plants are ready to harvest.   [6]

In another field, the farmer is growing the same variety of wheat, but is using a different fertilizer. The heights of these plants, F cm, are normally distributed with mean 98.6 and standard deviation d. The farmer finds the interquartile range to be 4.82 cm.

(d)   Find the value of d.   [3]